Lecture 7 Feb 19

Goals:

stacks

- Implementation of stack

applications

- Postfix expression evaluation
- Convert infix to postfix

Stack Overview

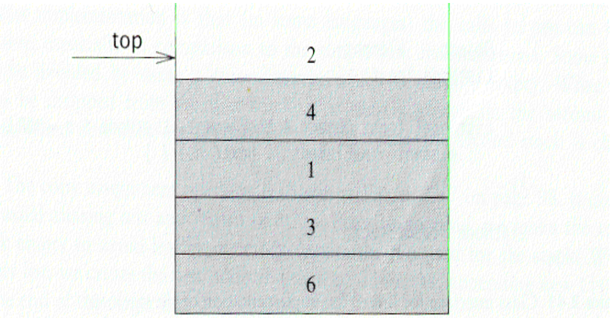
- Stack ADT
- Basic operations of stack
  - push, pop, top, isEmpty etc.

- Implementations of stacks using
  - Array
  - Linked list

- Application to arithmetic expression evaluation

Stack ADT

- A *stack* is a list in which insertion and deletion take place at the same end
  - This end is called *top*
  - The other end is called *bottom*



- Stacks are known as LIFO (Last In, First Out) lists.
  - The last element inserted will be the first to be retrieved

Push and Pop

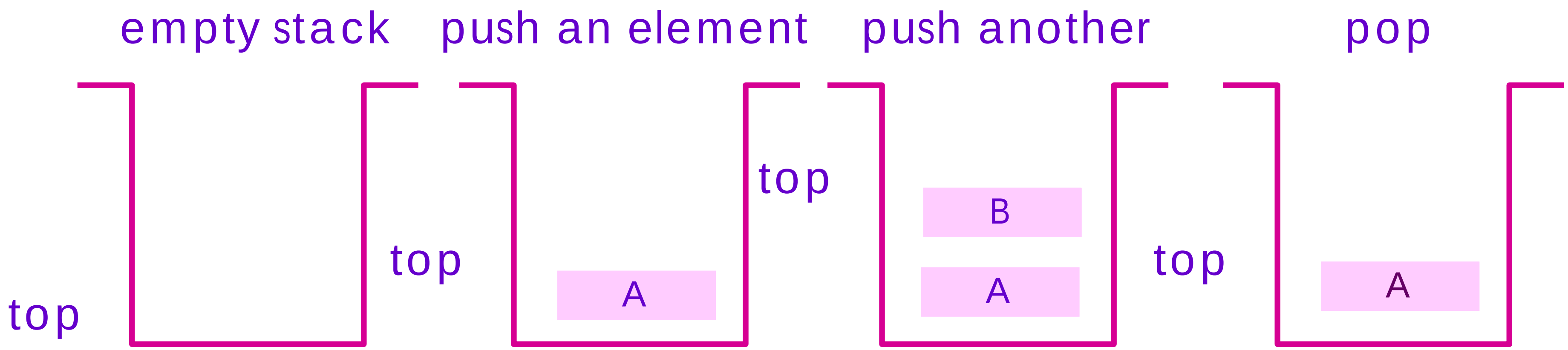
Primary operations: Push and Pop

Push

- Add an element to the top of the stack

Pop

- Remove the element at the top of the stack



Implementation of Stacks

Any list implementation could be used to implement a stack

- arrays (static: the size of stack is given initially)
- Linked lists (dynamic: never becomes full)

We will explore implementations based on array

Stack class

```
class Stack {
public:
      Stack(int size = 10);                    //
constructor
      ~Stack() { delete [] values; }      //
destructor
      bool IsEmpty() { return top == -1; }
      bool IsFull() { return top == maxTop; }
      double Top();   // examine, without popping
      void Push(const double x);
      double Pop();
      void DisplayStack();
private:
      int maxTop;       // max stack size = size - 1
      int top;          // current top of stack
      double* values;   // element array
};
```

Stack class

Attributes of Stack

- maxTop: the max size of stack
- top: the index of the top element of stack
- values: point to an array which stores elements of stack

Operations of Stack

- IsEmpty: return true if stack is empty, return false otherwise
- IsFull: return true if stack is full, return false otherwise
- Top: return the element at the top of stack
- Push: add an element to the top of stack
- Pop: delete the element at the top of stack
- DisplayStack: print all the data in the stack

Create Stack

The constructor of Stack

- Allocate a stack array of size. By default, size = 10.
- Initially top is set to -1. It means the stack is empty.
- When the stack is full, top will have its maximum value, i.e. size – 1.

```
Stack::Stack(int size /*= 10*/) {
      values      =     new double[size];
      top         =     -1;
      maxTop      =     size - 1;
}
```

Although the constructor dynamically allocates the stack array, the stack is still static. The size is fixed

Push Stack

```
void Push(const double x);
```

- Push an element onto the stack
- Note top always represents the index of the top element. After pushing an element, increment top.

```
void Stack::Push(const double x) {
      if (IsFull()) // if stack is full, print error
       cout << "Error: the stack is full." << endl;
      else
            values[++top]     =     x;
}
```

Pop Stack

`double Pop()`

- Pop and return the element at the top of the stack
- Don’t forgot to decrement top

```
double Stack::Pop() {
      if (IsEmpty()) { //if stack is empty, print error
       cout << "Error: the stack is empty." << endl;
            return -1;
      }
      else {
            return values[top--];
      }
}
```

Stack Top

`double Top()`

- Return the top element of the stack
- Unlike Pop, this function does not remove the top element

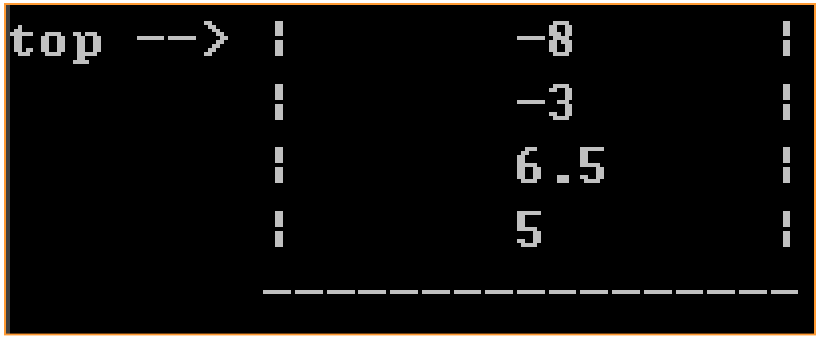
```
double Stack::Top() {
      if (IsEmpty()) {
       cout << "Error: the stack is empty." << endl;
            return -1;
      }
      else
            return values[top];
    }
```

Printing all the elements

`void DisplayStack()`

Print all the elements

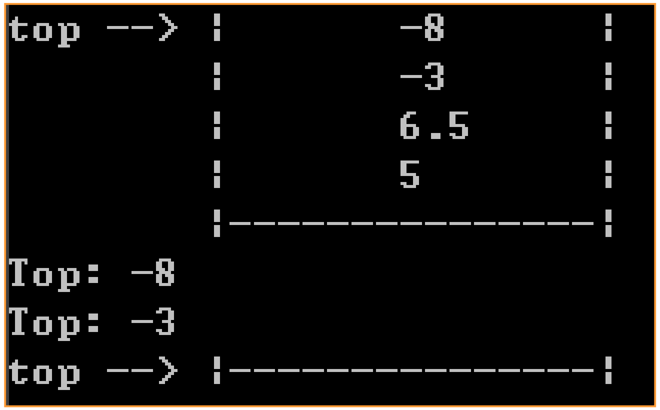
```
void Stack::DisplayStack() {
      cout << "top -->";
      for (int i = top; i >= 0; i--)
            cout << "\t|\t" << values[i] << "\t|" << endl;
      cout << "\t|---------------|" << endl;
}
```



```
top -->	|		-8		|
	|		-3		|
	|		6.5		|
	|		5		|
	-----------------
```

Using Stack

result



```
top --> ¦       -8      ¦
        ¦       -3      ¦
        ¦       6.5     ¦
        ¦       5       ¦
        ¦---------------¦
Top: -8
Top: -3
top --> ¦---------------¦
```

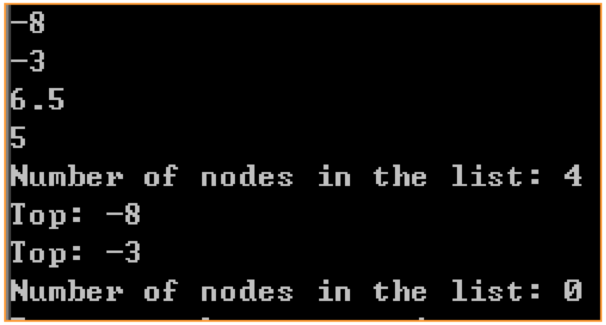
```
int main(void) {
      Stack stack(5);
      stack.Push(5.0);
      stack.Push(6.5);
      stack.Push(-3.0);
      stack.Push(-8.0);
      stack.DisplayStack();
      cout << "Top: " << stack.Top() << endl;

      stack.Pop();
      cout << "Top: " << stack.Top() << endl;
      while (!stack.IsEmpty()) stack.Pop();
      stack.DisplayStack();
      return 0;
}
```

Implementation based on Linked List

- Now let’s implement a stack based on a linked list
- To make the best out of the code of List, we implement Stack by inheriting List
  - To let Stack access private member head, we make Stack as a friend of List

```
class List {
public:
        List(void) { head = NULL; }           // constructor
        ~List(void);                          // destructor
        bool IsEmpty() { return head == NULL; }
        Node* InsertNode(int index, double x);
        int FindNode(double x);
        int DeleteNode(double x);
        void DisplayList(void);
private:
        Node* head;
        friend class Stack;
};
```

Implementation based on Linked List



```
-8
-3
6.5
5
Number of nodes in the list: 4
Top: -8
Top: -3
Number of nodes in the list: 0
```

```
class Stack : public List {
public:
        Stack() {}          // constructor
        ~Stack() {}         // destructor
        double Top() {
                if (head == NULL) {
                        cout << "Error: the stack is empty." << endl;
                        return -1;
                }
                else
                        return head->data;
        }
        void Push(const double x) { InsertNode(0, x); }
        double Pop() {
                if (head == NULL) {
                        cout << "Error: the stack is empty." << endl;
                        return -1;
                }
                else {
                        double val = head->data;
                        DeleteNode(val);
                        return val;
                }
        }
        void DisplayStack() { DisplayList(); }
};
```

Note: the stack implementation based on a linked list will never be full.

Array implementation versus linked list implementations

- push, pop, top are all constant-time operations in both array implementation and linked list implementation
  - For array implementation, the operations are performed in very fast constant time

Application 1: Balancing Symbols

To check that every right brace, bracket, and parentheses must correspond to its left counterpart

e.g. [( )]{ } is legal, but {[( ])} is illegal

Algorithm

(1) Make an empty stack.

(2) Read characters until end of file

   i. If the character is an opening symbol, push it onto the stack

   ii. If it is a closing symbol, then if the stack is empty, report an error

   iii. Otherwise, pop the stack. If the symbol popped is not the corresponding opening symbol, then report an error

(3) At end of file, if the stack is not empty, report an error

Application 2: Expression evaluation

Given an arithmetic expression such as:

x + y * (z + w)

(Given values assigned to x = 23, y = 12, z = 3 and w = -4)

What is the value of the expression?

Goal: Design a program that takes as input an arithmetic expression and evaluates it.

This task is an important part of compilers. As part of this evaluation, the program also needs to check if the given expression is correctly formed.

Example of bad expressions:

(3 + 12 * (5 – 3)

A + B * + C etc.

Postfix expression

Instead of writing the expression as A + B, we write the two operands, then the operator.

Example: a b c + *

It represents a*(b + c)

Question: What is the postfix form of the expression a+ b*c?

Postfix expression

Instead of writing the expression as A + B, we write the two operands, then the operator.

Example: a b c + *

It represents a*(b + c)

Question: What is the postfix form of the expression a+ b*c?

Answer: a b c * +

Algorithm for evaluating postfix expression

- Use a stack.
- Push operands on the stack.
- When you see an operator, pop off the top two elements of the stack, apply the operator, push the result back.
- At the end, there will be exactly one value left on the stack which is the final result.

Algorithm for evaluating postfix expression

- Use a stack.
- Push operands on the stack.
- When you see an operator, pop off the top two elements of the stack, apply the operator, push the result back.
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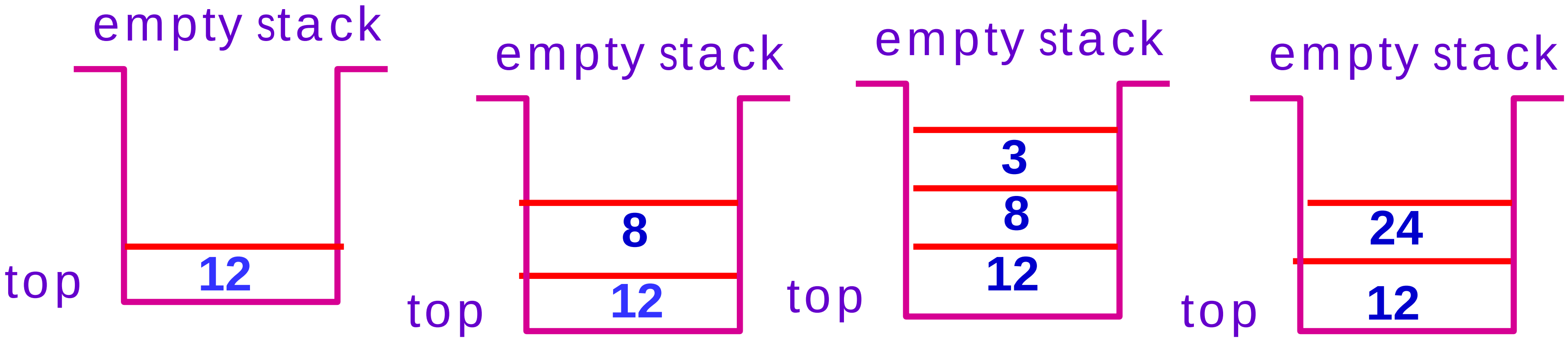
Example: 12 8 3 * +



empty stack
top
12

empty stack
top
8
12

empty stack
top
3
8
12

empty stack
top
24
12

Finally, the result 36 is pushed back on the stack.

Implementation of exp evaluation

token class:

```
class token {
 private: int op_type;
           double value;
  public:
     token(int x, int y) {
         op_type = x; op_value = y;
      }
     int get_op_type() {
        return op_type;
      }
      double get_value() {
        return value;
     }
     void set_op_type(int x) { op_type = x;
     }

     void set_value(double y) {value = y;
   }
 };
```

Op_type:

1 → +

2 → -

3 → *

4 → /

5 → **

6 → operand

–1 → token represents end of expression

op_value: value of the operand.

Input

Look at the main program:

```
int main(void) {
  string str = "908 100 200+   23 19 * +/ 123 *";
  Expr ex(str, 0);
  double rslt = ex.eval();
  cout << "The result of evaluation is " << rslt << endl;
  return 0;
};
```

```
C:\PROGRA~1\dm\bin>stack_eval
The result of evaluation is 151.539
```

There must be a space between successive operands. There need not be a space when an operand follows an operator, and after an operator. There can be more than one space after any token, including the last.

Implementation of exp evaluation

```
 double eval() {
       Stack st(MAX_SIZE);
       token tok = get_token();  //gets the next token
       while (tok.get_op_type() != -1) {
        if (tok.get_value() != 0)
          st.Push(tok.get_value());
        else {
          double opd2 = st.Pop();
          double opd1 = st.Pop();
          double op = apply(tok.get_op_type(), opd1, opd2);
          st.Push(op);
        }
        current++; tok = get_token();
       }
      double result = st.Pop(); return result;
    } // eval
}; // end Expr
```

Code for get_token

```
 token get_token() {
   token tok( -1, 0);
   if (current > exp.length() - 1)
        return tok;
  while (exp[current] == ' ') current++;
  if (current > exp.length() – 1) return tok;
  if (exp[current] == '+') tok.set_op_type(1);
   else if (exp[current] == '-') tok.set_op_type(2);
   else if (exp[current] == '/') tok.set_op_type(4);
   else if (exp[current] == '*') {
         if (exp[current+1] != '*') tok.set_op_type(3);
              else {tok.set_op_type(5); current++;}
         }
   else { // token is an operand
      double temp = 0.0;
      while (!(exp[current] == ' ') && !optr(exp[current])) {
                        temp= 10*temp+val(exp[current]); current++; }
               if (optr(exp[current])) current--;
               tok.set_op_type(6);
               tok.set_value(temp);
         }
     return tok;
} //end get_token
```