Data Compression

- Reduce the size of data.
  - Reduces storage space and hence storage cost.
    - Compression ratio = original data size / compressed data size
  - Reduces time to transmit and retrieve data.
  - Reduces the storage requirement. (particularly useful in embedded systems, network bridges, routers etc.)

Adapted from Sahni's *Data Structures and Applications* slides.

Lossless And Lossy Compression

- compressedData = compress(originalData)
- decompressedData =
  decompress(compressedData)
- When originalData = decompressedData, the compression is lossless.
- When originalData != decompressedData, the compression is lossy.

Lossless And Lossy Compression

- Lossy compressors generally obtain much higher compression ratios than do lossless compressors.
  - Say 100 vs. 2.
- Lossless compression is essential in applications such as text file compression.
- Lossy compression is acceptable in many imaging applications.
  - In video transmission, a slight loss in the transmitted video is not noticed by the human eye.

Text Compression

- Lossless compression is essential.

- Popular text compressors such as zip and Unix's compress are based on the LZW (Lempel-Ziv-Welch) method.

LZW Compression

- Character sequences in the original text are replaced by codes that are dynamically determined.
- The code table is not encoded into the compressed text, because it may be reconstructed from the compressed text during decompression.

LZW Compression

- Assume the letters in the text are limited to {a, b}.
  - In practice, the alphabet may be the 256 character ASCII set.
- The characters in the alphabet are assigned code numbers beginning at 0.
- The initial code table is:

code	0	1
key	a	b

LZW Compression

code	0	1
key	a	b

- Original text = *abababbabaabbabbaabba*
- Compression is done by scanning the original text from left to right.
- Find longest prefix p for which there is a code in the code table.
- Represent p by its code pCode and assign the next available code number to pc, where c is the next character in the text that is to be compressed.

LZW Compression

code	0	1	2
key	a	b	ab

- Original text = ababababbabaabbabbaabba
- p = a
- pCode = 0
- c = b
- Represent a by 0 and enter ab into the code table.
- Compressed text = 0

LZW Compression

code	0	1	2	3
key	a	b	ab	ba

- Original text = abababbabaabbabbaabba
- Compressed text = 0
  - p = b
  - pCode = 1
  - c = a
  - Represent b by 1 and enter ba into the code table.
  - Compressed text = 01

LZW Compression

code	0	1	2	3	4
key	a	b	ab	ba	aba

- Original text = abababbabaabbabbaabba
- Compressed text = 01
  - p = ab
  - pCode = 2
  - c = a
  - Represent ab by 2 and enter aba into the code table.
  - Compressed text = 012

LZW Compression

code	0	1	2	3	4	5
key	a	b	ab	ba	aba	abb

- Original text = abababbabaabbabbaabba
- Compressed text = 012
  - p = ab
  - pCode = 2
  - c = b
  - Represent ab by 2 and enter abb into the code table.
  - Compressed text = 0122

LZW Compression

code	0	1	2	3	4	5	6
key	a	b	ab	ba	aba	abb	bab

- Original text = abababbabaabbabbaabba
- Compressed text = 0122

- p = ba
- pCode = 3
- c = b
- Represent ba by 3 and enter bab into the code table.
- Compressed text = 01223

LZW Compression

code	0	1	2	3	4	5	6	7
key	a	b	ab	ba	aba	abb	bab	baa

- Original text = ababababbabaabbabbaabba
- Compressed text = 01223
- p = ba
- pCode = 3
- c = a
- Represent ba by 3 and enter baa into the code table.
- Compressed text = 012233

LZW Compression

code	0	1	2	3	4	5	6	7	8
key	a	b	ab	ba	aba	abb	bab	baa	abba

- Original text = ababababbabaabbabbaabba
- Compressed text = 012233
  - p = abb
  - pCode = 5
  - c = a
  - Represent abb by 5 and enter abba into the code table.
  - Compressed text = 0122335

LZW Compression

code	0	1	2	3	4	5	6	7	8	9
key	a	b	ab	ba	aba	abb	bab	baa	abba	abbaa

- Original text = abababbabaabbabbaabba
- Compressed text = 0122335
  - p = abba
  - pCode = 8
  - c = a
  - Represent abba by 8 and enter abbaa into the code table.
  - Compressed text = 01223358

LZW Compression

code	0	1	2	3	4	5	6	7	8	9
key	a	b	ab	ba	aba	abb	bab	baa	abba	abbaa

- Original text = abababbabaabbabbaabba
- Compressed text = 01223358
  - p = abba
  - pCode = 8
  - c = null
  - Represent abba by 8.
  - Compressed text = 012233588

Code Table Representation

code	0	1	2	3	4	5	6	7	8	9
key	a	b	ab	ba	aba	abb	bab	baa	abba	abbaa

- Dictionary.
  - Pairs are (key, element) = (key,code).
  - Operations are : get(key) and put(key, code)
- Limit number of codes to 2^{12}.
- Use a hash table.
  - Convert variable length keys into fixed length keys.
  - Each key has the form pc, where the string p is a key that is already in the table.
  - Replace pc with (pCode)c.

Code Table Representation

code	0	1	2	3	4	5	6	7	8	9
key	a	b	ab	ba	aba	abb	bab	baa	abba	abbaa

code	0	1	2	3	4	5	6	7	8	9
key	a	b	0b	1a	2a	2b	3b	3a	5a	8a

Implementation of LZW algorithm

```
void Compress()
{// Lempel-Ziv-Welch compressor.
   ChainHashTable<element, long> h(D);
   element e;
   for (int i = 0; i < alpha; i++) {// initialize
      e.key = i;
      e.code = i;
      h.Insert(e);
   }
   int used = alpha; // codes used
// input and compress
   unsigned char c;
   in.get(c);
   long pcode = c; // prefix code
```

Implementation of LZW algorithm

```
if (!in.eof()) {// file length is > 1
      do {// process rest of file
          in.get(c);
          if (in.eof()) break;  // finished
          long k = (pcode << ByteSize) + c;
          // see if code for k in dictionary
          if (h.Search(k, e)) pcode = e.code;  // yes
          else {// k not in table
                output(pcode);
                if (used < codes) // create new code
                {e.code = used++;
                 e.key = (pcode << ByteSize) | c;
                 h.Insert(e);}
                pcode = c;}
       } while (true);
      output(pcode);
      if (status) {c = LeftOver << excess;
                   out.put(c);}
      }
   out.close(); in.close();
}
```

LZW Decompression

code	0	1
key	a	b

- Original text = abababbabaabbabbaabba
- Compressed text = 012233588

- Convert codes to text from left to right.
- 0 represents a.
- Decompressed text = a
- pCode = 0 and p = a.
- p = a followed by next text character (c) is entered into the code table.

LZW Decompression

code	0	1	2
key	a	b	ab

- Original text = abababbabaabbabbaabba
- Compressed text = 012233588

- 1 represents b.
- Decompressed text = ab
- pCode = 1 and p = b.
- lastP = a followed by first character of p is entered into the code table.

LZW Decompression

code	0	1	2	3
key	a	b	ab	ba

- Original text = abababbabaabbabbaabba
- Compressed text = 012233588

- 2 represents ab.
- Decompressed text = abab
- pCode = 2 and p = ab.
- lastP = b followed by first character of p is entered into the code table.

LZW Decompression

code	0	1	2	3	4
key	a	b	ab	ba	aba

- Original text = ababababbabaabbabbaabba
- Compressed text = 012233588

  - 2 represents ab
  - Decompressed text = ababab.
  - pCode = 2 and p = ab.
  - lastP = ab followed by first character of p is entered into the code table.

LZW Decompression

code	0	1	2	3	4	5
key	a	b	ab	ba	aba	abb

- Original text = abababbabaabbabbaabba
- Compressed text = 012233588

- 3 represents ba
- Decompressed text = abababba.
- pCode = 3 and p = ba.
- lastP = ab followed by first character of p is entered into the code table.

LZW Decompression

code	0	1	2	3	4	5	6
key	a	b	ab	ba	aba	abb	bab

- Original text = abababbabaabbabbaabba
- Compressed text = 012233588

- 3 represents ba
- Decompressed text = abababbaba.
- pCode = 3 and p = ba.
- lastP = ba followed by first character of p is entered into the code table.

LZW Decompression

code	0	1	2	3	4	5	6	7
key	a	b	ab	ba	aba	abb	bab	baa

- Original text = ababababbabaabbabbaabba
- Compressed text = 012233588

- 5 represents abb
- Decompressed text = abababbabaabb.
- pCode = 5 and p = abb.
- lastP = ba followed by first character of p is entered into the code table.

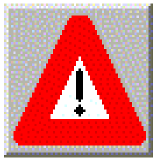
LZW Decompression

code	0	1	2	3	4	5	6	7	8
key	a	b	ab	ba	aba	abb	bab	baa	abba

- Original text = abababbabaabbabbaabba
- Compressed text = 012233588

- 8 represents ???

- When a code is not in the table, its key is lastP followed by first character of lastP.



- lastP = abb
- So 8 represents abba.

LZW Decompression

code	0	1	2	3	4	5	6	7	8	9
key	a	b	ab	ba	aba	abb	bab	baa	abba	abbaa

- Original text = abababbabaabbabbaabba
- Compressed text = 012233588

- 8 represents abba
- Decompressed text = abababbabaabbabbaabba.
- pCode = 8 and p = abba.
- lastP = abba followed by first character of p is entered into the code table.

Code Table Representation

code	0	1	2	3	4	5	6	7	8	9
key	a	b	ab	ba	aba	abb	bab	baa	abba	abbaa

- Dictionary.
  - Pairs are (key, element) = (code, what the code represents) = (code, codeKey).
  - Operations are : get(key) and put(key, code)
- Keys are integers 0, 1, 2, …
- Use a 1D array codeTable.
  - codeTable[code] = codeKey.
  - Each code key has the form pc, where the string p is a code key that is already in the table.
  - Replace pc with (pCode)c.

Time Complexity

- Compression.
  - O(n) expected time, where n is the length of the text that is being compressed.
- Decompression.
  - O(n) time, where n is the length of the decompressed text.