Lecture 10 Feb 28

Goals:

- hashing
    - dictionary operations
    - general idea of hashing
    - hash functions
    - chaining
    - closed hashing

Dictionary operations

- search
- insert
- delete

Applications:

- employees in a company
- books in a library
- web pages (e.g. in web searching)
- geometric shapes in a graphics application

Dictionary operations

- search
- insert
- delete

	ARRAY		LINKED LIST	
	sorted	unsorted	sorted	unsorted
Search	O(log n)	O(n)	O(n)	O(n)
Insert	O(n)	O(1)	O(n)	O(n)
delete	O(n)	O(n)	O(n)	O(n)

comparisons and data movements combined (Assuming keys can be compared with <, > and = outcomes)

Exercise: Create a similar table separately for data movements and for comparisons.

Performance goal for dictionary operations:

O(n) is too inefficient.

Goal is to achieve each of the operations

(a) in O(log n) on average

(b) (b) worst-case O(log n)

(c) constant time O(1) on average.

Data structure that achieve these goals:

(a) binary search tree

(b) balanced binary search tree (AVL tree)

(c) hashing. (but worst-case is O(n))

Hashing

- An important and widely useful technique for implementing dictionaries
- Constant time per operation (on the average)
- Worst case time proportional to the size of the set for each operation (just like array and linked list implementation)

General idea

U = Set of all possible keys: (e.g. 9 digit SS #)

If $n = |U|$ is not very large, a simple way to support dictionary operations is:

map each key e in U to a unique integer h(e) in the range 0 .. n – 1.

Boolean array H[0 .. n – 1] to store keys.

General idea

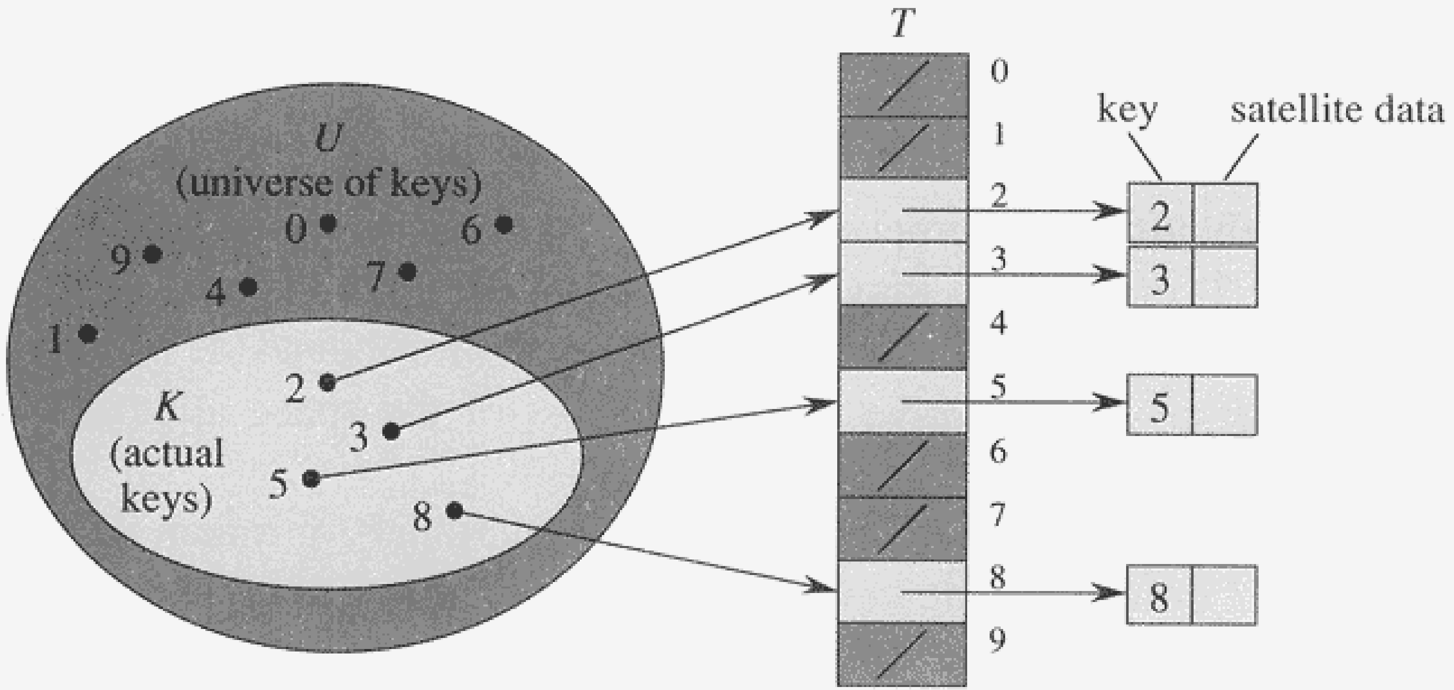


Figure 11.1 Implementing a dynamic set by a direct-address table T. Each key in the universe $U = \{0, 1, \ldots, 9\}$ corresponds to an index in the table. The set $K = \{2, 3, 5, 8\}$ of actual keys determines the slots in the table that contain pointers to elements. The other slots, heavily shaded, contain NIL.

Ideal case not realistic

- U the set of all possible keys is usually very large so we can't create an array of size n = |U|.
- Create an array H of size m much smaller than n.

- Actual keys present at any time will usually be smaller than n.
- mapping from U -> {0, 1, …, m – 1} is called hash function.

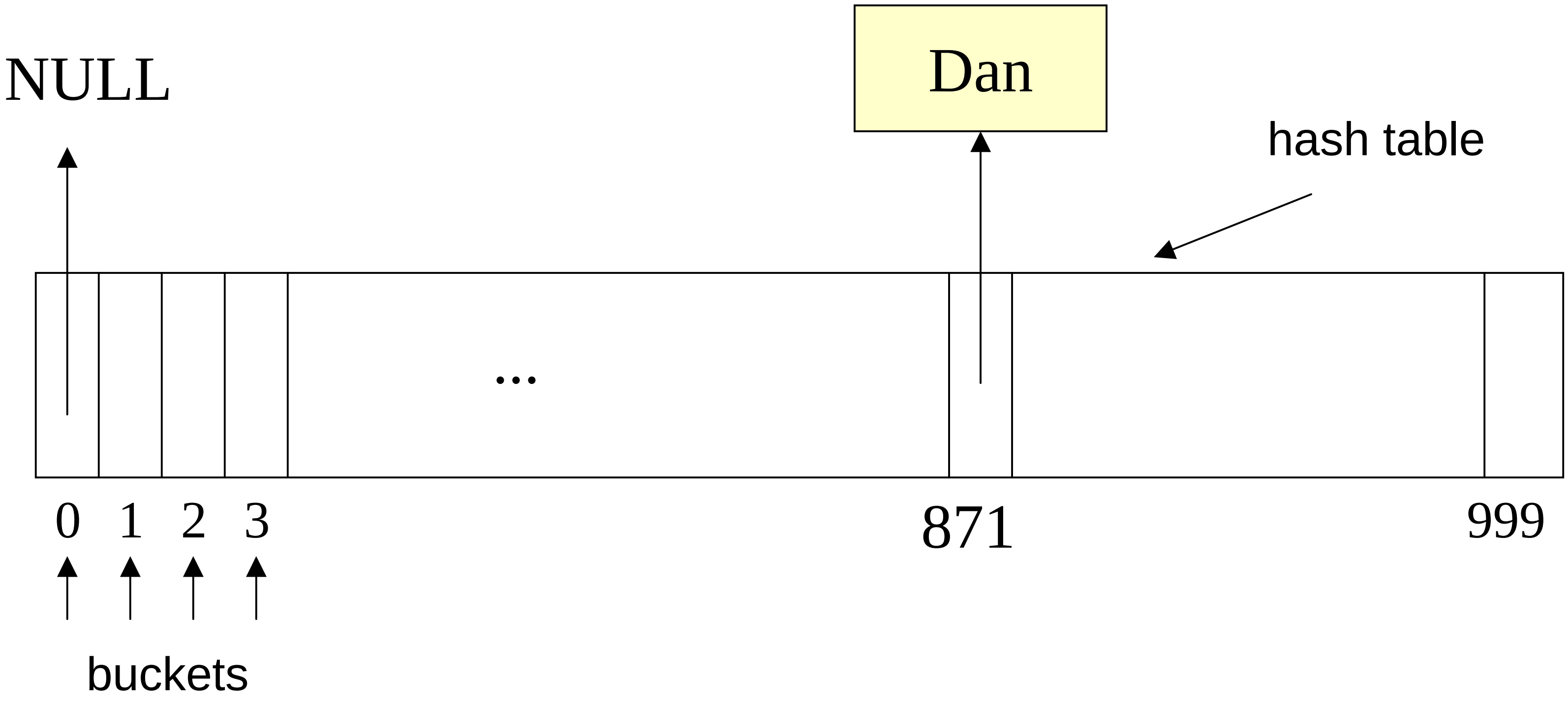
Example: D = students currently enrolled in courses, U = set of all SS #'s, hash table of size = 1000

Hash function h(x) = last fthree digits.

Example (continued)

Insert Student “Dan” SS# = 1238769871

h(1238769871) = 871

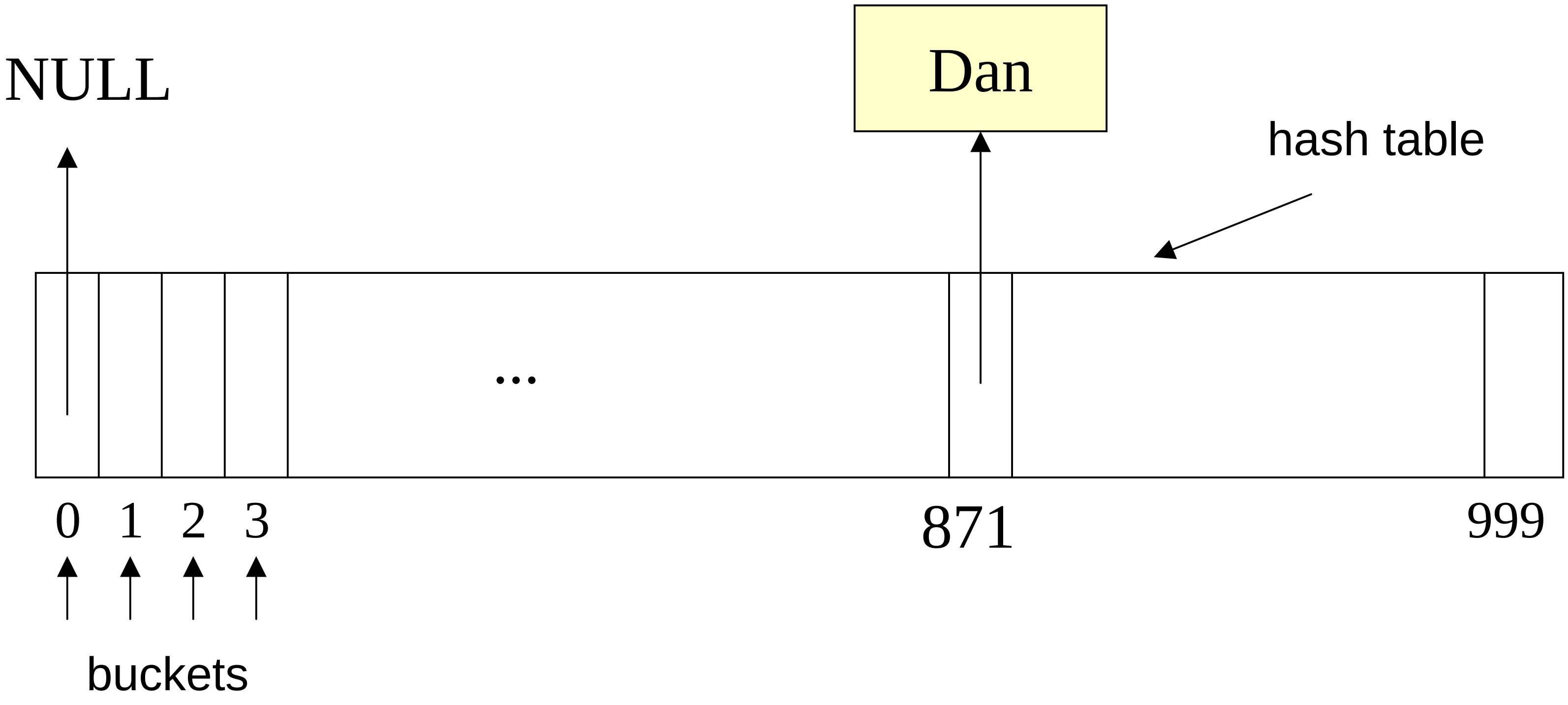


Example (continued)

Insert Student “Tim” SS# = 1872769**871**

h(1238769871) = 871, same as that of Dan.

Collision



Hash Functions

If $h(k_1) = \beta = h(k_2)$: k_1 and k_2 have collision at slot β

There are two approaches to resolve collisions.

Collision Resolution Policies

Two ways to resolve:

(1) Open hashing, also known as separate chaining

(2) Closed hashing, a.k.a. open addressing

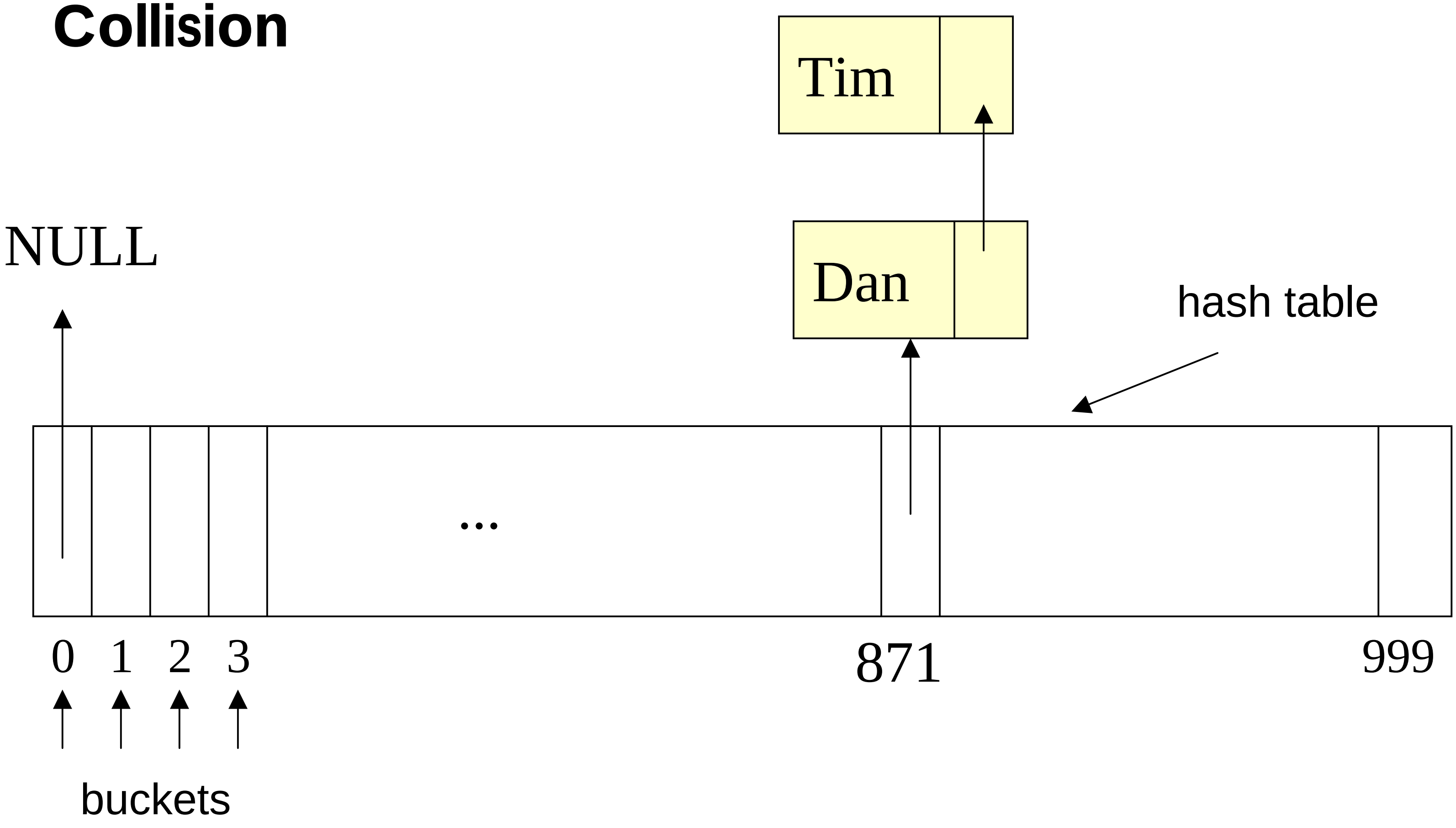
Chaining: keys that collide are stored in a linked list.

Previous Example:

Insert Student “Tim” SS# = 1872769**871**

h(1238769871) = 871, same as that of Dan.

Collision



Open Hashing

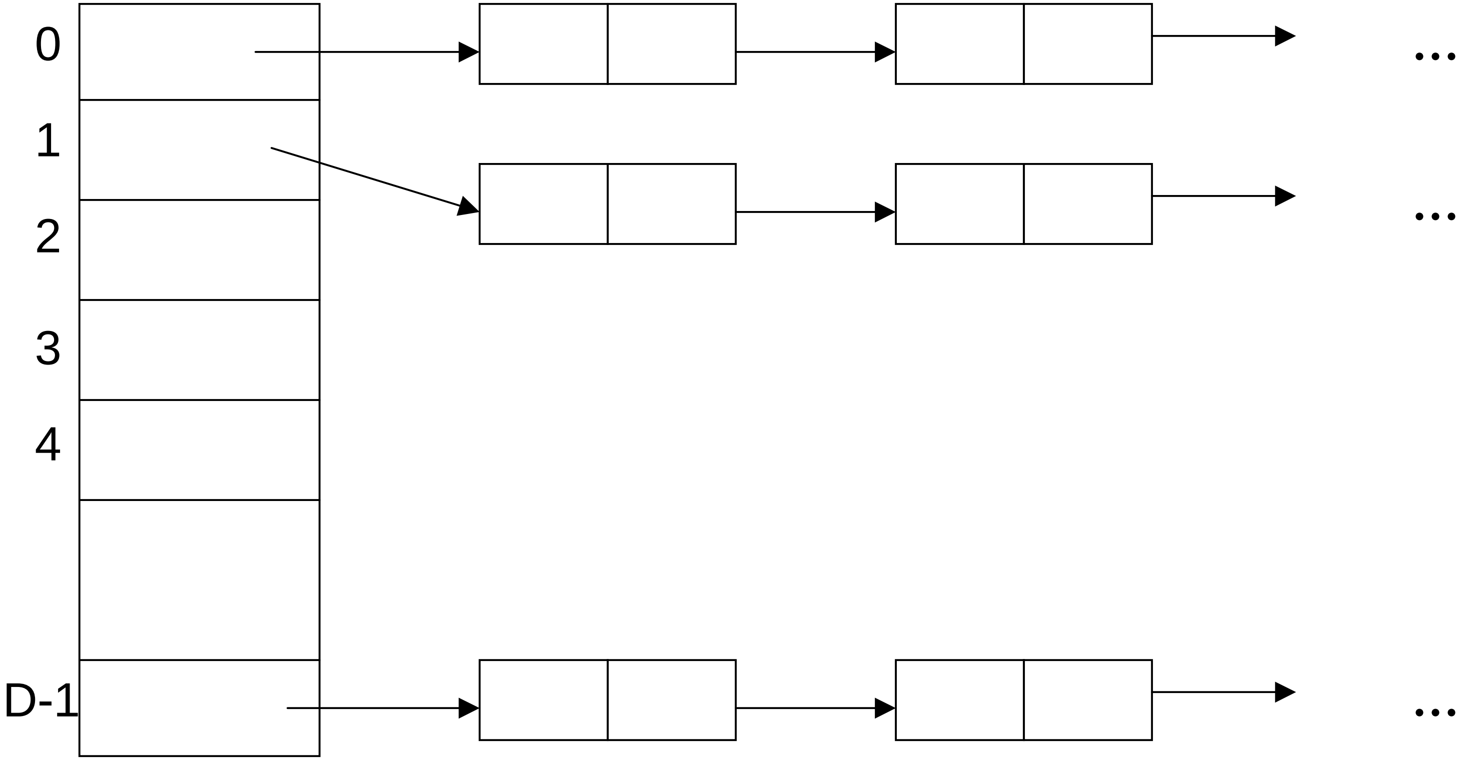
The hash table is a pointer to the head of a linked list

All elements that hash to a particular bucket are placed on that bucket's linked list

Records within a bucket can be ordered in several ways

- by order of insertion, by key value order, or by frequency of access order

Open Hashing Data Organization



Implementation of open hashing - search

```
bool contains( const HashedObj & x )
  {
   list<HashedObj> whichList = theLists[ myhash( x ) ];
   return find( whichList.begin( ), whichList.end( ), x ) !=
        whichList.end( );
  }
```

Find is a function in the STL class algorithm. Code for find is described below:

```
template<class InputIterator, class T>
InputIterator find ( InputIterator first, InputIterator last,
      const T& value ) {
   for ( ;first!=last; first++)
     if ( *first==value ) break;
   return first; }
```

Implementation of open hashing - insert

```
bool insert( const HashedObj & x )
{
  list<HashedObj> whichList = theLists[ myhash( x ) ];
  if( find( whichList.begin( ), whichList.end( ), x ) !=
          whichList.end( ) )
  return false;
  whichList.push_back( x );
  return true;
}
```

The new key is inserted at the end of the list.

Implementation of open hashing - delete

```
bool remove( const HashedObj & x )
    {
        list<HashedObj> & whichList = theLists[ myhash( x ) ];
        list<HashedObj>::iterator itr =
             find( whichList.begin( ), whichList.end( ), x );

        if( itr == whichList.end( ) )
            return false;

        whichList.erase( itr );
        --currentSize;
        return true;
   }
```

Choice of hash function

A good hash function should:

- be easy to compute
- distribute the keys uniformly to the buckets
- use all the fields of the key object.

Example: key is a string over {a, ..., z, 0, ... 9, _ }
Suppose hash table size is n = 10007.

(Choose table size to be a prime number.)

Good hash function: interpret the string as a number to base 37 and compute mod 10007.

h(“word”) = ? “w” = 23, “o” = 15, “r” = 18 and “d” = 4.

h(“word”) = (23 * 37 + 15 * 37 + 18 * 37 + 4) % 10007

Computing hash function for a string

Horner’s rule:

```
int hash( const string & key )
{
    int hashVal = 0;

    for( int i = 0; i < key.length( ); i++ )
        hashVal = 37 * hashVal + key[ i ];

    return hashVal;
}
```