

Project # 5 image compression
(final-version)

Due: May 18, 2008

Problem Statement

A given image in bmp format will be compressed by representing it using a quad-tree representation. The details of this representation and the compression and decompression algorithms are described below. The input will be a bit map image, the outputs will be a compressed representation of the image (which is a string over the alphabet {0, 1, 2, #}) and the decompressed image. An integer parameter d , a tolerance factor (d in $[0, 255]$) that describe the degree of compression will also be specified. Since the algorithm is a lossy one, the output image will be generally not the same the input image. The larger value of d will result in greater loss of quality. You will also implement an algorithm to search an individual pixel in the compressed representation of the image. The search will be interactive and when the user specifies the row and column values of a pixel, the program will output the RGB value of that pixel (in the compressed image).

Goals

- Learn about tree representation which is a pointer-based tree representation.
- Learn and implement an algorithm for compression and decompression using a recursive algorithm.
- Implement an algorithm to access individual pixels in the compressed image.
- Experimentally determine the trade-off between the compression achieved and the quality of the recovered image.

Quad-trees

We will describe a data structure called a quad-tree. A node in this tree represents a subregion of the image. This node can be a leaf node in which case all the pixels in the region are identical (or nearly identical in case of lossy representation). If not, the region is divided into four quadrants and each region is (recursively) represented by a subtree. A tree built in this way, by selecting a single square region to corresponding to the root of T and then creating descendants according to the procedure above, is called a *quad-tree*. See Figure 1.

Notice that the node at the blue level of the tree corresponds to the cell of a 1x1 grid over the root square; the nodes at the purple level of the tree correspond to cells of a 2x2 grid over the root square; the nodes at the red level of the tree correspond to cells of a 4x4 grid over the root square; the nodes at the black level of the tree correspond to cells of an 8x8 grid over the root square; and so on.

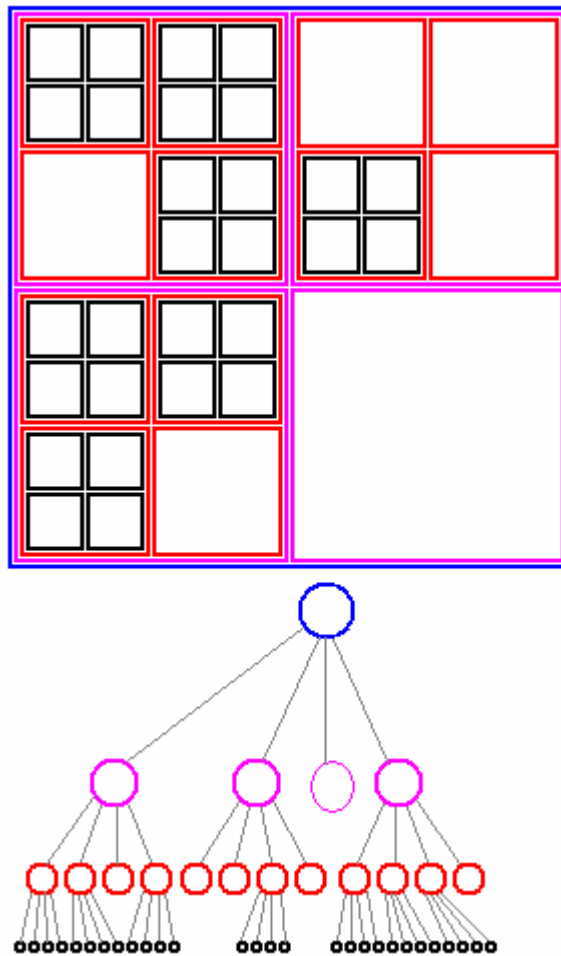


Figure 1. Quad-tree, with pointers ordered NW, NE, SE, SW.

In Figure 1, the 34 leaves of the tree correspond to the 34 unequal squares into which the root square is split by the diagram.

Compression algorithm

The basic idea behind the compression algorithm is as follows: The algorithm takes a parameter d which is an integer between 0 and 100. We create the quad-tree recursively starting with the original image at the root and dividing it into image of half its size and recursively creating the tree corresponding to the four subimages. When we reach a node such that each (RGB) color component of the pixels differ by at most d , then this node becomes a leaf node whose color components are the average of the corresponding component of all the pixels in that subregion. Each leaf node is assigned a 24-bit integer (RGB value). The internal nodes are assigned a value of 2. A preorder traversal on the tree will give a string over the alphabet $\{0,1,2\}$ which

is the compressed representation of the image. Shown below is an example. On the left is a simple black and white image, On the right is the quad-tree.

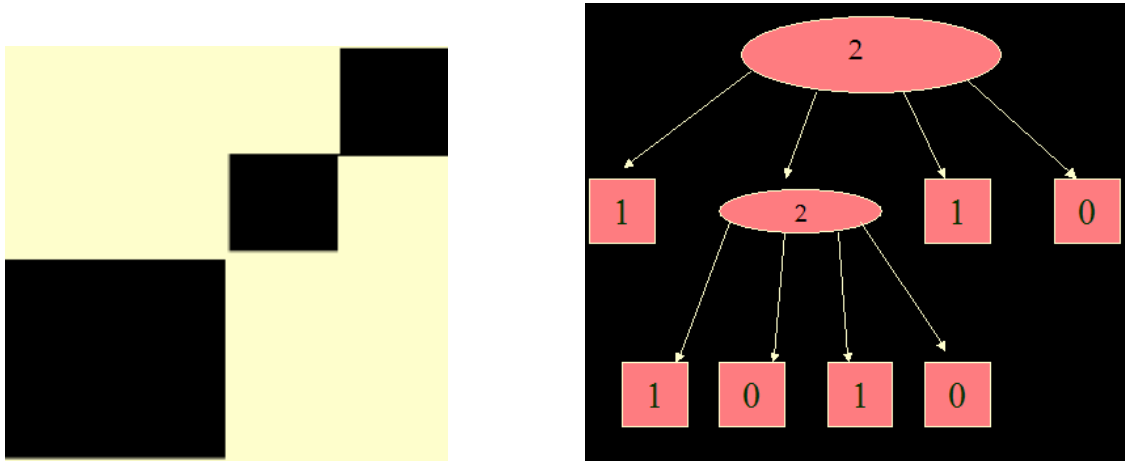


Figure 2: An image and its tree representation. In the figure on the right, 0 represents [0,0,0] and 1 represents [255,255,255].

The encoding of the quad-tree would be $\langle \text{size} \rangle \# \langle \text{tree} \rangle$ where size is the size of the image, followed by the symbol #, followed by the **preorder** traversal of the tree. Thus, suppose the image on the left is 128 by 128 pixels. Then its encoding as represented by the quad-tree on the right is $10^7 \# 21^{24} 21^{24} 0^{24} 1^{24} 0^{24} 1^{24} 0^{24}$. In the above, a^b represents $aaaa \dots a$ (repeated b times). We will assume that the dimension of the image is of the form $2^k \times 2^k$ for some k .

Input: An image I , and a parameter d . Assume: $\text{width}(I) = \text{height}(I)$ is a power of 2.

Output: Compressed representation of the image.

Step 1: If the image is small (say has size 4 by 4) represent it by a single node whose label is the average color of the 16 pixels. Return a pointer T to this node.

Step 2. Else (i.e., $\text{width}(I) = \text{height}(I) \geq 8$) if a single color c dominates I (i.e., all the pixels are in the interval $[c, c + d - 1]$) for some c for each color component, then create a single node and assign the average color component to this node and return a pointer T to the node.

Step 3: Else, divide the image into four quadrants $NW(I)$, $NE(I)$, $SE(I)$ and $SW(I)$. Recursively apply the algorithm on each quadrant to get the subtrees $T1$, $T2$, $T3$ and $T4$. Create a new node N , and make $T1, \dots, T4$ its children. Return a pointer T to N .

Step 4: To get the string representation of the quad-tree, perform an preorder traversal.

Decompression algorithm

This is the inverse operation in which the string is converted back to the image. It will be assumed that the input contains a valid encoding. The details of this algorithm are as follows. The algorithm takes as input a string and constructs the quadtree representation.

Input: string $s[0..n-1]$

Output: T = quadtree representation of s .

Step 1: let $s = s1 \#s2$.

Step 2: Return $T = \text{convert}(s2, 0, s1)$.

Convert is a recursive procedure described below:

```
qtree convert(string s, int& posn, int size)
if (s[posn] != 2) {
    create a node N with dimension = size, R = s[posn .. posn + 7], G = s[posn + 8 .. posn +
    15] and B = s[posn + 16 .. posn + 23], set the children links to null. posn += 24; Return N }
else {
    T1 = convert(s, posn+1, size/4);
    T2 = convert(s, posn, size/4);
    T3 = convert(s, posn, size/4);
    T4 = convert(s, posn, size/4);
    Create a node N. Set N->dimension = size, N->NW = T1, N->NE = T2, N->SE = T3 and
    N->SW = T4. Return N. }
}
```

Search for individual pixels

The problem is to access the individual pixel of an image from the compressed representation. One way to do this is will be to decompress the image and use the standard EasyBMP function to access the pixel. However, here we will implement an algorithm directly to access a given pixel. The search procedure is very similar to searching in a binary search tree.

Input: string s that represents the image in compressed format, p , q where p represents the column number and q represents the row number of the pixel. Recall that the $(0,0)$ represents the top left corner pixel.

Output: RGB representation of the pixel, e.g., $[212, 0, 34]$.

Algorithm:

Step 1: convert s into a quad-tree representation T .

Step 2: If s is a leaf, then output the label of s and stop.

Step 3: Using the values (p,q) , determine which of the four children contains the pixel (p,q) and recursively continue the search on that subtree.

Quadtree class

A quad-tree class with the data members and member functions is suggested below. However, you need not follow this suggestion exactly.

```
class qtree {

    int dimension; RGBApixel data;

    qtree* NW, NE, SE, SW;

    qtree(RGBApixel p); // constructor creates a leaf node with p as pixel.

    RGBApixel check(BMP& image, int xval, int yval, int width, int d, bool& check1) ;

    // this procedure returns true (false) via the variable check1 if  $\max C - \min C \leq d$  for  $c = R, G$ 
    //and B in the subimage whose upper-left corner is  $(xval, yval)$  and R as width and height.  $d =$ 
    //tolerance. If check1 is true, then the average pixel is returned.

    build(BMP& image, int xval, int yval, int width, int d);

    // returns a quadtree representation of the subimage whose upper-left corner is
    //  $(xval, yval)$  and R = width and height,  $d =$  tolerance

    RGBApixel search(int xval, int yval) // outputs the color values of the specified pixel.

    string compress( ); // string representation of the quad-tree

    void uncompress(string comp_img);

    // converts the string representation comp_img into quad-tree

}
```

Details of submission

When your program is run, a dialog similar to the one shown below should occur. The last integer parameter is the d value.

```
% project5 image_in image_out 12
Enter 1 to access a pixel, enter 0 to quit
1
Enter the x and y values of the pixel you want to access
134 67
```

Pixel inaccessible, image size is 128 x 128

Enter 1 to access a pixel, enter 0 to quit

1 12 67

The colors of the pixel at (1 12, 67) is: R = 0, G = 200, B = 20

Enter 1 to access a pixel, enter 0 to quit

0

%